

CALT-68-1965
DOE RESEARCH AND
DEVELOPMENT REPORT

Evidence for Non-perturbative String Symmetries*

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*Presented at the International Congress of Mathematical Physics
Satellite Conference “Topology, Strings, and Integrable Models”
Paris, July 1994*

Abstract

String theory appears to admit a group of discrete field transformations – called S dualities – as exact non-perturbative quantum symmetries. Mathematically, they are rather analogous to the better-known T duality symmetries, which hold perturbatively. In this talk the evidence for S duality is reviewed and some speculations are presented.

*Work supported in part by the U.S. Dept. of Energy under Grant No. DE-FG03-92-ER40701.

1. Introduction

Non-compact global symmetries are a pervasive feature of supergravity theories. Typically, the group G is realized nonlinearly on scalar fields that parametrize the homogeneous space G/H , where H is the maximal compact subgroup of G . The first example of this phenomenon, with $G = SL(2, \mathbb{R})$ and $H = U(1)$, was uncovered in 1976 in a version of $N = 4$, $D = 4$ supergravity by Cremmer, Ferrara, and Scherk.¹ Curiously, this particular example corresponds precisely to the low-energy effective field theory associated with the most studied example of S duality in string theory – the toroidally compactified heterotic string. An analogous non-compact E_7 symmetry was found in $N = 8$, $D = 4$ supergravity by Cremmer and Julia in 1978,² and many other examples were worked out thereafter.³

In 1990, Font Ibañez, Lüster, and Quevedo proposed⁴ that an $SL(2, \mathbb{Z})$ subgroup of the $SL(2, \mathbb{R})$ of Ref. 1 should be an exact symmetry of the heterotic string toroidally compactified (à la Narain⁵) to four dimensions. This discrete symmetry group is called S duality, because the $N = 1$ superfield (containing the axion and dilaton) that parametrizes $SL(2, \mathbb{R})/U(1)$ is often called S . That S duality should be an exact symmetry was a bold conjecture, since it implies, as a special case, an electric-magnetic duality in which the coupling constant is inverted ($g_{el} \rightarrow g_{mag} \propto 1/g_{el}$). Thus, if true, it has implications beyond perturbation theory. On the other hand, string theory is well understood only in perturbation theory. A non-perturbative formulation is still lacking. Thus, when it first appeared, the proposal of Font *et al.*, seemed to me (and probably to others, too) to be intriguing but impossible to test. As I will describe, this is not the case. Impressive non-trivial tests of S duality have been formulated and verified. Thus my point of view now is that S duality and its extensions should be explored as broadly as possible as part of a program that may someday culminate in a non-perturbative formulation of quantum string theory. Indeed, the symmetry might well play a central role in the construction.

In string theory, as in classical supergravity, S duality is realized by field transformations. As such, it is a statement about the theory and not about classical solutions or quantum ground states. Any particular solution will generically result in complete spontaneous breaking of the symmetry. In special cases (corresponding to orbifold points of the moduli space) a small subgroup, such as \mathbb{Z}_2 or \mathbb{Z}_3 , may remain unbroken. Thus S duality is analogous to a broken gauge symmetry. Though it is a discrete symmetry group, it is believed (mostly by analogy with T duality) to be a local symmetry of string theory in the sense that field configurations that are related by group transformations should be identified and counted just once in the path integral. Note that this definition does not imply that group elements are functions of space-time. This is just as well since the fundamental string action – whatever it may be – is unlikely to make explicit reference to space-time. The existence of a

space-time should be only a (large-distance) feature of a certain class of solutions.

For a variety of reasons, I suspect that the $SL(2, \mathbb{Z})$ group is a small piece of a much larger symmetry group. The complete group might be the hyperbolic Kac–Moody group E_{10} ,⁶ or a discrete subgroup $E_{10}(\mathbb{Z})$,⁷ or some continuous group or supergroup containing $E_{10}(\mathbb{Z})$. (Recall that T dualities are special cases of continuous group transformations.)

Another approach to uncovering the underlying symmetry in string theory, pioneered by Gross and Mende,⁸ is the study of string amplitudes at very high energy. A more algebraic approach to obtaining much of the same information has been developed recently by Moore.⁹ The correspondence between the symmetries found by such methods and the S and T duality symmetries that I will discuss, is not fully understood. Since the analyses of Refs. 8 and 9 are based on perturbation theory, they would seem capable of revealing T duality symmetries only. However, it may be possible to extract more information than that. Shenker¹⁰ has presented evidence that string theory has non-perturbative phenomena that are of form $\exp[-c/g]$ in addition to the familiar instanton-like effects that go as $\exp[-c/g^2]$. It would be very interesting to see how this meshes with S duality. Another related issue that needs to be better understood is the structure of the high-temperature phase of string theory – above the Hagedorn transition.¹¹ This, too, requires non-perturbative information. The answer is apparently essential for an understanding of how string theory overcomes the much-discussed entropy and information-loss problems associated with black holes.¹²

2. Two Kinds of Duality

T duality or “target space duality” is a discrete symmetry of various string theories that holds order by order in perturbation theory. (For a review see Ref. 13.) In the simplest case of compactification on a circle the group is Z_2 and the transformation corresponds to inversion of the radius ($R \rightarrow \alpha'/R$). Of course, R is determined by the value of scalar field (a T modulus) and the symmetry is realized as a field transformation. For any value other than $R = \sqrt{\alpha'}$, the symmetry is spontaneously broken. The generalization to toroidal compactification of the heterotic string (à la Narain) has been studied in detail by many authors. In this case no supersymmetry is broken and (for $d = 4$) there are 132 scalar fields that live on the homogeneous space $M_0 = O(6, 22)/O(6) \times O(22)$. However, the T duality group is $G_T = O(6, 22; \mathbb{Z})$ and the moduli space is, therefore, $M = M_0/G_T$. The 132 scalar fields belong to 22 abelian $N = 4$ gauge multiplets.

The toroidally compactified heterotic string also contains two additional scalar fields – called the axion χ and the dilaton ϕ – which belong to the $N = 4$ supergravity multiplet. This supergravity theory is precisely the one studied in Ref. 1, where χ

and ϕ were shown to parametrize $SL(2, \mathbb{R})/U(1)$. To show how this works, let us introduce a complex scalar field.

$$\lambda = \chi + ie^{-\phi} = \lambda_1 + i\lambda_2 . \quad (1)$$

In terms of λ , the symmetry is realized by linear fractional transformations

$$\lambda \rightarrow \frac{a\lambda + b}{c\lambda + d} , \quad (2)$$

where $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{R})$. The classical value of λ is $\langle \lambda \rangle = \frac{\theta}{2\pi} + \frac{8\pi i}{g^2}$, where θ is the vacuum angle and g is the coupling constant.[†] When instanton effects are taken into account the Peccei-Quinn symmetry $\chi \rightarrow \chi + b$, which corresponds to the $SL(2, \mathbb{R})$ subgroup given by matrices $\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$, is broken to the discrete subgroup for which b is an integer. This subgroup and the inversion $\lambda \rightarrow -1/\lambda$ generate the discrete group $SL(2, \mathbb{Z})$ or $PSL(2, \mathbb{Z})$.

Mathematically, S and T duality are quite analogous in the 4D low-energy effective field theory (EFT) even though their implications for string theory are dramatically different. This analogy was one of the original motivations for proposing that S duality should also be a symmetry. Let me briefly describe the bosonic sector of the EFT and mention how the symmetries are realized. (For more details see Ref. 14.) The massless bosonic fields are the metric tensor $g_{\mu\nu}$, the axion-dilaton field λ , (where χ is related to the antisymmetric tensor field $B_{\mu\nu}$ by a duality transformation), 28 abelian gauge fields A_μ^a , and 132 moduli M^{ab} parametrizing $O(6, 22)/O(6) \times O(22)$. These are the only massless fields at generic points in the classical moduli space. At the singular points, which we ignore here, there are more. The moduli M^{ab} are conveniently described as a symmetric 28×28 matrix belonging to the group $O(6, 22)$:

$$M^T = M, \quad M^T L M = L \quad (3)$$

$$L = \begin{pmatrix} 0 & I_6 & 0 \\ I_6 & 0 & 0 \\ 0 & 0 & I_{22} \end{pmatrix} . \quad (4)$$

The classical action with the fields given above is

$$S = (const.) \int d^4x \sqrt{-g} \left(R - \frac{g^{\mu\nu}}{2\lambda_2^2} \partial_\mu \lambda \partial_\nu \bar{\lambda} \right)$$

[†]Recall that $N = 4$ Yang–Mills theories have vanishing β function, so that θ and g^2 are well-defined independent of scale. This feature presumably holds for heterotic strings in $N = 4$ symmetric backgrounds, as well.

$$+ \frac{1}{8} g^{\mu\nu} \text{tr}(\partial_\mu M L \partial_\nu M L) + L_{gauge} \Big) , \quad (5)$$

where

$$L_{gauge} = \frac{\lambda_1}{4} F_{\mu\nu}^a L_{ab} \tilde{F}^{b\mu\nu} - \frac{\lambda_2}{4} (L M L)_{ab} F_{\mu\nu}^a F^{\mu\nu b} . \quad (6)$$

As usual, $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$ and $\tilde{F}^{\mu\nu} = \frac{1}{2\sqrt{-g}} \epsilon^{\mu\nu\rho\lambda} F_{\rho\lambda}$. Under T duality, the transformation rules are

$$M \rightarrow \Omega M \Omega^T, A_\mu \rightarrow \Omega A_\mu , \quad (7)$$

where $\Omega^T L \Omega = L$ and $g_{\mu\nu}$ and λ are invariant. Each term in the action is manifestly invariant under these transformations. The group here is $O(6, 22)$, but in string theory when one includes the Kaluza–Klein and winding-mode excitations it is broken to the discrete subgroup $G_T = O(6, 22; \mathbb{Z})$.

Under S duality one has

$$\lambda \rightarrow \frac{a\lambda + b}{c\lambda + d} \quad (8)$$

$$F_{\mu\nu}^a \rightarrow (c\lambda_1 + d) F_{\mu\nu}^a + c\lambda_2 (M L)_{ab} \tilde{F}_{\mu\nu}^b , \quad (9)$$

while $g_{\mu\nu}$ and M^{ab} are invariant. This is a symmetry of the first three terms of the action, but not of L_{gauge} . However, one can show that it is a symmetry of the equations of motion that follow from S . This suggests there may be a different action giving rise to the same equations of motion that has S duality symmetry. Such an action was constructed by Sen and me,¹⁵ but I will not present it here. I will simply remark that we could make S and T duality simultaneously manifest by sacrificing manifest general coordinate invariance. It is not known whether it is possible to introduce auxiliary fields so as to realize all three symmetries off-shell at the same time. The problem is analogous to that for supersymmetry for which auxiliary fields can sometimes be found and sometimes not. If one insists on off-shell closure of the algebra, it would be natural to include supersymmetry at the same time. Of course, our real interest is string theory, for which off-shell issues still seem remote (and maybe even irrelevant).

3. The Soliton Spectrum

The 28 $U(1)$ gauge fields A_μ^a give rise to 28 sets of electric and magnetic charges. A convenient way to define them is to assume that space-time is asymptotically flat and use the asymptotic behavior of the field strengths:

$$F_{0i}^a \sim \frac{q_{el}^a}{r^3} x^i \quad \tilde{F}_{0i}^a \sim \frac{q_{mag}^a}{r^3} x^i . \quad (10)$$

As explained in Ref. 14, the allowed charges are then controlled by the asymptotic values of the moduli ($\lambda \sim \lambda^{(0)}$ and $M_{ab} \sim M_{ab}^{(0)}$) and a pair of vectors α_0^a, β_0^a belonging to the Narain lattice, which is an even self-dual Lorentzian lattice of signature (6,22). The formulas are

$$q_{el}^a = \frac{1}{\lambda_2^{(0)}} M_{ab}^{(0)} (\alpha_0^b + \lambda_1^{(0)} \beta_0^b), \quad q_{mag}^a = L_{ab} \beta_0^b. \quad (11)$$

A nice feature of these formulas is that they automatically incorporate the Dirac–Schwinger–Zwanziger–Witten quantization rules (*i.e.*, the quantization condition for dyons with a θ angle).

The central charges that appear in the $N = 4$ supersymmetry algebra are determined in terms of the electric and magnetic charges. Moreover, the mass of a state is bounded below by its central charges. This bound, known as the Bogomol’nyi bound implies that an $N = 4$ multiplet with charges given by (α_0^a, β_0^a) must satisfy

$$(\text{Mass})^2 \geq \frac{1}{16} (M^{(0)} + L)_{ab} (\alpha_0^a \beta_0^a) \mathcal{M}^{(0)} \begin{pmatrix} \alpha_0^b \\ \beta_0^b \end{pmatrix}, \quad (12)$$

where the 2×2 matrix $\mathcal{M}^{(0)}$ is the asymptotic value of

$$\mathcal{M} = \frac{1}{\lambda_2} \begin{pmatrix} 1 & \lambda_1 \\ \lambda_1 & |\lambda|^2 \end{pmatrix}. \quad (13)$$

This matrix transforms under S duality in a manner analogous to the way M transforms under T duality. Thus, the Bogomol’nyi bound is manifestly invariant under both S and T duality provided that (α_0^a, β_0^a) is a (2,28) representation of $G_S \times G_T$, which is indeed the case. Note that since α_0^a and β_0^a belong to a lattice, it is essential that these are the discrete groups $G_S = SL(2, \mathbb{Z})$ and $G_T = O(6, 22; \mathbb{Z})$.

This is very nice, but it certainly doesn’t prove that $G_S \times G_T$ is a symmetry of the toroidally compactified heterotic string. T duality works perturbatively and is well understood, but how can we prove S duality without knowing non-perturbative string theory? As yet, we cannot prove it, but we can subject the conjecture to some non-trivial tests. Specifically, if we focus on states that saturate the bound in eq. (12), then we have “short” representations of the $N = 4$ algebra. Generically, a representation would have dimension $2^8 = 256$, but when the bound is saturated it has dimension $2^4 = 16$. (The gauge multiplets with vanishing masses and charges are examples.) The analysis is analogous to that for the Poincaré group, which admits short representations for massless fields. The essential fact, pointed out long ago by Witten and Olive,¹⁶ is that such states can receive no quantum corrections – perturbative or nonperturbative – to their masses so long as the supersymmetry remains unbroken. Accepting that, we need only establish that the degeneracies of

such 16-dimensional representations, $N_{16}(\vec{\alpha}_0, \vec{\beta}_0)$, are $SL(2, \mathbb{Z})$ invariant. Note that because the vacuum breaks S and T duality spontaneously, the states saturating the Bogomol'nyi bound do not form degenerate multiplets of these groups. Eq. (12) is invariant only when $\mathcal{M}^{(0)}$ and $M^{(0)}$ are transformed together with (α_0^a, β_0^a) . If specific values of $\mathcal{M}^{(0)}$ and $M^{(0)}$ leave a subgroup of $G_S \times G_T$ unbroken, then degenerate multiplets correspond to representations of that subgroup.

Let us now explore which states in the elementary string spectrum saturate the Bogomol'nyi bound. First of all, such states carry electric charges only (associated with internal momentum and winding-mode excitations). Thus, absorbing $M^{(0)}$ in the definition of the lattice, eq. (12) simplifies to

$$(\text{Mass})^2 = \frac{1}{16\lambda_2^{(0)}} \hat{\alpha}^a (I + L)_{ab} \hat{\alpha}^b = \frac{1}{8\lambda_2^{(0)}} (\hat{\alpha}_R)^2, \quad (14)$$

where $\hat{\alpha} \cdot \hat{\alpha} = \hat{\alpha}_L \cdot \hat{\alpha}_L - \hat{\alpha}_R \cdot \hat{\alpha}_R$. ($\hat{\alpha}_L$ is 22-dimensional and $\hat{\alpha}_R$ is 6-dimensional. They correspond to the left-moving and right-moving internal momenta of the string.) Now we should compare the free string spectrum, which is given by

$$(\text{Mass})^2 = \frac{1}{4\lambda_2^{(0)}} \left[\frac{1}{2} (\hat{\alpha}_L)^2 + N_L - 1 \right] = \frac{1}{4\lambda_2^{(0)}} \left[\frac{1}{2} (\hat{\alpha}_R)^2 + N_R - \delta \right]. \quad (15)$$

N_L and N_R represent left-moving and right-moving oscillator excitations. The parameter δ is 1/2 in the NS sector and 0 in the R sector. Alternatively, it is simply 0 in the GS formulation. The factor of $(\lambda_2^{(0)})^{-1}$ appears because the mass is computed with respect to the canonically normalized Einstein metric. It does not appear if one uses the string metric, which differs by a dilaton-dependent Weyl rescaling. The Einstein metric is more natural in the present context, because it is invariant under S duality transformations. Comparing formulas, one sees that the Bogomol'nyi bound is saturated provided that $N_R = \delta$ (which gives 8 bosonic and 8 fermionic right-moving modes – the short representation of $N = 4$) and $N_L = 1 + \frac{1}{2} \hat{\alpha} \cdot \hat{\alpha}$. Thus, if $\hat{\alpha} \cdot \hat{\alpha} = 2n - 2$, for a non-negative integer n , then there is a short $N = 4$ multiplet for every solution of $N_L = n$. These states are only “electrically” charged. The challenge is to find their predicted S duality partners. Specifically, every elementary string excitation of the type we have just described ($\vec{\alpha} = \vec{\ell}, \vec{\beta} = 0$) should have magnetically charged partners with $\vec{\alpha} = a\vec{\ell}$ and $\vec{\beta} = c\vec{\ell}$. Since a and c are elements of an $SL(2, \mathbb{Z})$ matrix, they are relatively prime integers. If such states exist, one can use the Bogomol'nyi bound to prove that they are absolutely stable. This is important since the formula for the mass, which is supposed to be exact, is real.

Sen has investigated the partners of electrically charged states with $\vec{\ell} \cdot \vec{\ell} = -2$ ($N_L = 0$). He has explained that S duality partners with $c = 1$ can be identified with BPS monopole solutions (and their dyonic generalizations) of the EFT. These

solutions saturate the bound, of course. Thus, as we have explained, they should persist with exactly this mass in the complete quantum string theory. For $c > 1$, Sen argues that one should examine multi-BPS dyon bound states. Specifically, he shows that the prediction of S duality is that each multi-BPS dyon moduli space should admit a unique normalizable harmonic form. Poincaré duality would give a second one unless it is self-dual or anti-self-dual. He then constructs such an anti-self-dual form explicitly for the case of $c = 2$,¹⁷ providing the most non-trivial test of S duality yet. Progress toward extending this result to $c > 2$ is attributed to G. Segal (unpublished).

The next case to consider is $\vec{\ell} \cdot \vec{\ell} = 0$, corresponding to $N_L = 1$. This case probes stringy structure, which makes it especially interesting. Since states with $N_L = 1$ are given by a single oscillator excitation $\alpha_{-1}^\mu |0\rangle$, one should find states with helicity ± 1 once each and ones with helicity 0 22 times. Solitons with $\vec{\ell} \cdot \vec{\ell} = 0$ have been obtained¹⁸ by wrapping five-branes around the torus. They are called “ H monopoles” because the gauge fields in question arise from dimensional reduction of $H = dB$ (rather than the metric or 10-dimensional gauge fields). The question then is again the cohomology of the corresponding moduli space. The relevant part of the moduli space $\hat{\mathcal{M}}$ is four-dimensional and hyper-Kähler. A normalizable harmonic (p, q) form would give a state with left-moving helicity $\frac{1}{2}(p - q)$. In their first paper on the subject, Gauntlett and Harvey noted that K3 is a hyper-Kähler space with precisely the desired cohomology. However, they soon realized that it does not have the required $SO(3)$ isometry, and so they suggested¹⁹ that actually $\hat{\mathcal{M}} = R^4/\mathbb{Z}_2$. The cohomology of this space only gives 8 of the 24 desired states, so they suggested that the 16 remaining spin-0 states might arise from a stringy twisted sector. This example still needs more study. In particular, the noncompactness of $\hat{\mathcal{M}}$ gives rise to infra-red issues that need to be clarified.

4. Concluding Discussion

We have shown that S duality is a classical symmetry of $N = 4, D = 4$ supergravity coupled to abelian gauge multiplets. The hypothesis that it is an exact symmetry of string theory leads to specific testable predictions for the soliton spectrum of the heterotic string compactified on a torus. Several magnetically charged states have been shown to occur with precisely the predicted properties. Because of the essential way in which $N = 4$ supersymmetry and the Bogomol’nyi bound are used, it was important to carry out these tests for a special class of vacua (ones with unbroken $N = 4$ supersymmetry and abelian unbroken gauge symmetries). However, the underlying symmetry is realized in terms of field transformations, and so the particular choice of vacuum shouldn’t be of fundamental importance, just a matter of

convenience.

Recently, Girardello *et al.* and Vafa and Witten have examined the implications of S duality for $N = 4$ Yang–Mills,²⁰ and Seiberg and Witten have examined $N = 2$ Yang–Mills.²¹ I will not review those beautiful works here, but simply make a few remarks. First of all, in these theories the field λ is replaced by its value $\langle \lambda \rangle = \frac{\theta}{2\pi} + i\frac{8\pi}{g^2}$. Therefore, S duality is no longer a symmetry realized by field transformations, but rather a rule stating that the theory for one value $\langle \lambda \rangle$ is equivalent to the one for a transformed value – provided one suitably redefines the electric and magnetic charges at the same time. Such a duality was conjectured long ago by Montonen and Olive.²² In the $N = 4$ case, Vafa and Witten show that the partition function $Z(\lambda)$ transforms under $SL(2, \mathbb{Z})$ as a modular form whose weight is determined by the Euler characteristic of the four-manifold on which the theory is defined. (In the full string theory, I would expect other fields to give a compensating contribution so that the λ integral is modular invariant.) The $N = 2$ case is more subtle since $\langle \lambda \rangle$ is scale dependent when the theory is asymptotically free. The appropriate object to compute turns out to be the quantum moduli space, which is described by an elliptic (or hyperelliptic²³) curve. The reader is referred to Ref. [21] for more information.

In conclusion, our understanding of S duality is progressing rapidly. It should be helpful in the search for a more fundamental formulation of string theory. Also, its implications for vacua with $N = 1$ supersymmetry could be of phenomenological interest.

REFERENCES

1. E. Cremmer, J. Scherk, and S. Ferrara, Phys. Lett. **74B** (1978) 61; E. Cremmer and J. Scherk, Phys. Lett. **74B** (1978) 341.
2. E. Cremmer and B. Julia, Phys. Lett. **80B** (1978) 48; Nucl. Phys. **B159** (1979) 141.
3. A. Salam and E. Sezgin, eds., *Supergravities in Diverse Dimensions*, reprints in 2 vols., World Scientific (1989).
4. A. Font, L. Ibañez, D. Lüst, and F. Quevedo, Phys. Lett., **B249** (1990) 35; S.J. Rey, Phys. Rev. **D43** (1991) 526.
5. K. Narain, Phys. Lett. **B169** (1986) 41; K. Narain, H. Sarmadi, and E. Witten, Nucl. Phys. **B279** (1987) 369.
6. R. Gebert and H. Nicolai, hep-th/9406175.
7. C. Hull and P. Townsend, hep-th/9410167.
8. D. Gross, Phys. Rev. Lett. **60** (1988) 1229; D. Gross and P. Mende, Phys. Lett. **197B** (1987) 129; Nucl. Phys. **B303** (1988) 407; P. Mende and H. Ooguri, Nucl. Phys. **B339** (1990) 641.
9. G. Moore, hep-th/9310026 and hep-th/9404025.

10. S. Shenker, p. 191 in proceedings of 1990 Cargèse workshop *Random Surfaces and Quantum Gravity*.
11. J. Atick and E. Witten, Nucl. Phys. **B310** (1988) 291.
12. L. Susskind, hep-th/9309145, L. Susskind and J. Uglum, Phys. Rev. **D50** (1994) 2700; A. Dabholkar, hep-th/9408098 and hep-th/9409158.
13. A. Giveon, M. Porrati, and E. Rabinovici, Phys. Rept. **244** (1994) 77.
14. A. Sen, Int. J. Mod. Phys. **A9** (1994) 3707.
15. J. Schwarz and A. Sen, Nucl. Phys. **B404** (1993) 109.
16. E. Witten and D. Olive, Phys. Lett. **B78** (1978) 97.
17. A. Sen, Phys. Lett. **B329** (1994) 217.
18. J. Harvey, J. Gauntlett, and J. Liu, Nucl. Phys. **B409** (1993) 363; R. Khuri, Phys. Lett. **B259** (1991) 261.
19. J. Gauntlett and J. Harvey, hep-th/9407111; J. Gauntlett, private communication.
20. L. Girardello, A. Giveon, M. Porrati, and A. Zaffaroni, Phys. Lett. **B334** (1994) 331; C. Vafa and E. Witten, hep-th/9408074.
21. N. Seiberg and E. Witten, hep-th/9407087 and hep-th/9408099.
22. C. Montonen and D. Olive, Phys. Lett. **B72** (1977) 117; H. Osborne, Phys. Lett. **B83** (1979) 321.
23. A. Klemm, W. Lerche, S. Yankielowicz, and S. Theisen, hep-th/9411048; P. Argyres and A. Faraggi, hep-th/9411057.